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Heavy ion beam lifetimes at relativistic and ultrarelativistic colliders

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Abstract

The effects of higher order corrections in ultra-relativistic nuclear collisions are considered. It is found that higher order contributions are small at low energy, large at intermediate energy and small again at very high energy. An explanation for this effect is given. This means that the Weizsacker-Williams formula is a good approximation to use in calculating cross sections and beam lifetimes at energies relevant to RHIC and LHC.

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Heavy ion and relativistic heavy ion accelerators have so far been constructed with an incident beam and a fixed target. The first heavy ion *collider*, named the Relativistic Heavy Ion Collider (RHIC) [1], is currently under construction at Brookhaven National Laboratory. This machine will collide heavy ions all the way from the proton up to Pb nuclei with an energy per beam of 100 GeV/nucleon. This is equivalent to a fixed target accelerator with a beam energy of 21.7 TeV/nucleon. The Large Hadron Collider (LHC) is currently being planned for the purposes of proton-proton collisions. However LHC will also be capable of accelerating heavy ions. One of the main purposes in constructing these two relativistic heavy ion colliders is to undertake a search for the quark gluon plasma phase.

Heavy nuclear beams installed in heavy ion colliders will only have a finite lifetime due mainly to beam-beam Coulomb dissociation and electron pair production. In designing heavy ion colliders it is important to have good estimates for the various beam lifetimes [1]. Such an estimate for U-U beams installed in RHIC have been carried out [2]. However in that work the effects of higher order electromagnetic processes [3] was left out. Such effects are known to be small (about 4%) at low energy [4]. However these effects have been found to be larger (about 12%) at higher energies [3]. Thus the question immediately arises as to how big these effects might become at the very high energies planned for RHIC and LHC. In particular, if the effects are quite large then previous estimates of the beam lifetimes [1, 2] might be incorrect. This may have a significant effect on accelerator operations.

Generally the higher order cross sections are smaller than the Weizsacker-Williams cross section. In multiple electromagnetic processes the excitation of the single photon giant dipole resonance is depleted by multiple higher order electromagnetic processes, which can excite double and multiple giant resonances, generally lying at double and multiple integral values of the energy of the single dipole state. The Weizsacker-Williams calculation assumes that all of the photons excite only the single giant dipole resonance state.

Calculations were performed for the Coulomb dissociation cross sections, σ_{WW}, using the Weizsacker-Williams method [5] for Uranium-Uranium (UU) collisions at a variety of energies. The calculations were repeated for the cross sections including higher order corrections, σ_N, as discussed in [3]. The percentage difference, $100(\sigma_{WW} - \sigma_N)/\sigma_{WW}$, is then calculated. The results for fission of one of the U nuclei are presented in Figure 1. As discussed above, it is seen that the higher order effects are small at low energy and then grow as the energy is increased. However the surprising feature is that

as the energy is increased even further, the higher order effect starts getting smaller again. For RHIC and LHC energies the percentage difference is less than 2%. This is good news for calculations of beam lifetimes, because it means that the old Weizsacker-Williams calculations [1, 2] remain correct. It has been found previously [6] that other corrections, such as Rutherford bending and electric quadrupole effects, can also be neglected at high energy. Thus at the very high energies of RHIC and LHC one can essentially ignore all corrections (including higher order corrections), and just rely on the basic Weizsacker-Williams formula. The calculations represented in Figure 1 are for fission, but similar results were also found for other Coulomb dissociation reactions such as single and double nucleon removal and also for other nuclei.

Further analysis revealed the source of the behavior seen in Figure 1. In performing a Weizsacker-Williams calculation one folds and integrates the virtual photon spectrum with the photonuclear cross section as in

$$\sigma_{WW} = \int N(E)\sigma(E)dE \tag{1}$$

where σ is the nucleus-nucleus Coulomb dissociation cross section, $N(E)$ is the virtual photon spectrum as a function of virtual photon energy E, and $\sigma(E)$ is the photonuclear cross section. The virtual photon spectrum is a smoothly dropping function of virtual photon energy [5], while the photonuclear cross section is a bell shaped curve centered on a fixed photon energy. As the heavy ion energy is increased the amount of "overlap" between the virtual photon spectrum and the photonuclear cross section steadily increases. In impact parameter space this cross section is written [3]

$$\sigma = 2\pi \int P(b)bdb \tag{2}$$

where $P(b) = PWW(b)$ is the Weizsacker-Williams probability given by

$$PWW(b) = \Phi(b) \tag{3}$$

where

$$\Phi(b) = \int n(E,b)\sigma(E)dE/E \tag{4}$$

with $n(E,b)$ being the impact parameter dependent virtual photon spectrum [3]. Thus if $PWW(b)$ is used for $P(b)$ in equation (2) then one reproduces the numerical values obtained from (1). The higher order cross section is calculated with equation (2) except that

$$PN(b) = \Phi(b)e^{-\Phi(b)} \tag{5}$$

is used for the probability $P(b)$. This Poisson probability distribution results from the statistical independence of the emission of a number N of photons emitted from a classical current source [7].

Now to explain Figure 1. The probabilities, multiplied by b, (since it is b x *probability* that gets integrated in (2)) are plotted in Figures 2 - 4. At very low energies $\Phi(b)$ will be very small, so that $\Phi(b)$ and $\Phi(b)e^{-\Phi(b)}$ will be approximately the same leading to equality of the Weizsacker-Williams cross section σ_{WW} and the higher order cross section σ_N. This is seen in Figure 2, and is reflected in Figure 1 at the low energy end where the Weizsacker-Williams and higher order cross sections are approximately the same. As the energy is increased however, $\Phi(b)$ increases and $\Phi(b)e^{-\Phi(b)}$ starts getting smaller than $\Phi(b)$, as shown in Figure 3, meaning that σ_N is smaller than σ_{WW} which explains the rising part of Figure 1 between 10^{-2} GeV/nucleon and 10^0 GeV/nucleon. Remember that it is the areas under the curves in Figures 2 - 4 that gives the cross sections according to (2).

Figure 4 shows the probabilities (times b) at very high projectile energy. Even though there is still a difference between the probability curves, however now both curves extend out to much larger impact paramteres where the differences are much smaller. (High energies probe larger impact parameters.) This results in the total areas under the two curves in Figure 4 being much closer in value to each other than the two curves in Figure 3. (This was seen by explicit computation.) This means that for the case of Figure 4 the resulting cross sections (calculated from (2)) are in fact quite close to each other. This explains the small percentage difference value in Figure 1 at high energy. Consequently, the curve in Figure 1 has reached some maximum value (near 1 GeV/nucleon) and then starts falling as the energy increases. These various effects then explain the behaviour seen in Figure 1.

At very high energy then, one can safely use just the Weizsacker-Williams formula for calculating beam lifetimes, even for the heaviest of nuclei where one might have thought that higher order effects might have been important. As an application of this a Uranium beam lifetime is calculated for LHC. The method of calculation is the same as described previously [2]. The beam parameters [8] are listed in Table 1, where k is the number of beam intersections, L_0 is the initial luminosity, B is the number of bunches and N_B is the number of particles per bunch. Cross sections and lifetime results are listed in Table 2, where some previous results [2, 9] for RHIC are also listed for comparison. (The greatest uncertainty in these calculations is the pair production cross section $\sigma_{e^+e^-}$ [9]. Also in [2], the value for $\sigma_{e^+e^-}$ should

have been 85 barn.) Table 2 shows that Uranium beams will live sufficiently long so that they could be installed at RHIC and LHC.

In conclusion, it has been shown that higher order contributions to Coulomb dissociation are small at low energy, large at intermediate energy and then small again at very high energy. Consequently, the use of the Weizsacker-Williams formula in calculating cross sections and beam lifetimes is expected to be accurate at RHIC and LHC energies.

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Beam parameter	RHIC	LHC
k	6	1
L_0 (cm^{-2} s^{-1})	9.2×10^{26}	2×10^{27}
B	57	608
N_B	1.1×10^{9}	9.4×10^{7}
energy/nucleon/beam (GeV)	100	2760

Table 1: Beam Parameters

		σ (barns)			τ (hours)
	fission	1N	2N	e^+e^-	
RHIC	33	48	31	85	11
LHC	56	80	52	135	17

Table 2: Weizsacker-Williams Results

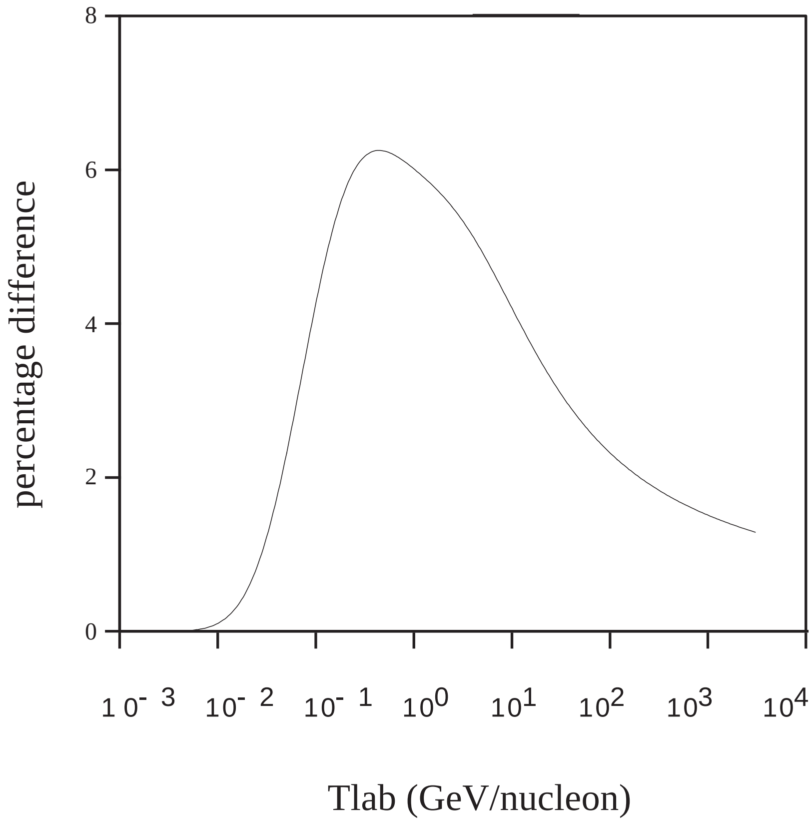

Figure 1

Percentage difference between Weizsacker-Williams and higher order cross sections for fission in UU collisions as a function of projectile kinetic energy for an equivalent fixed target accelerator. The RHIC energy (21,700 GeV/nucleon) and the LHC energy (16,237 TeV/nucleon) lie beyond the far right of the graph.

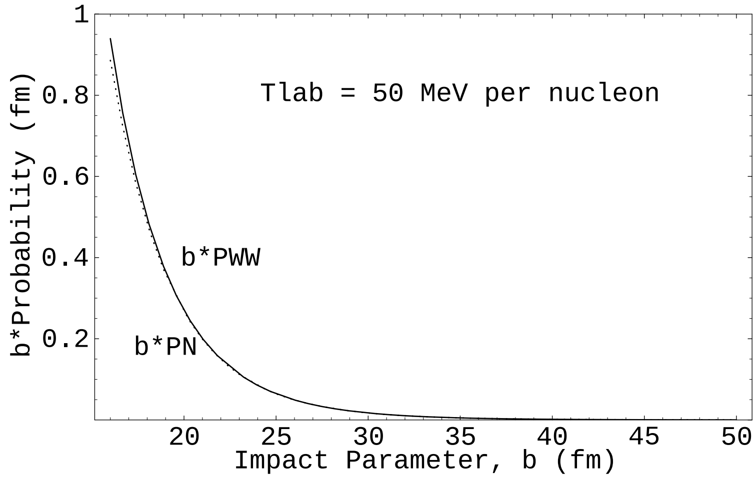

Figure 2

Impact parameter multiplied by probability as a function of impact parameter for Weizsacker-Williams and higher order theory for a projectile energy of 50 MeV/nucleon. The area under each curve gives the total cross section.

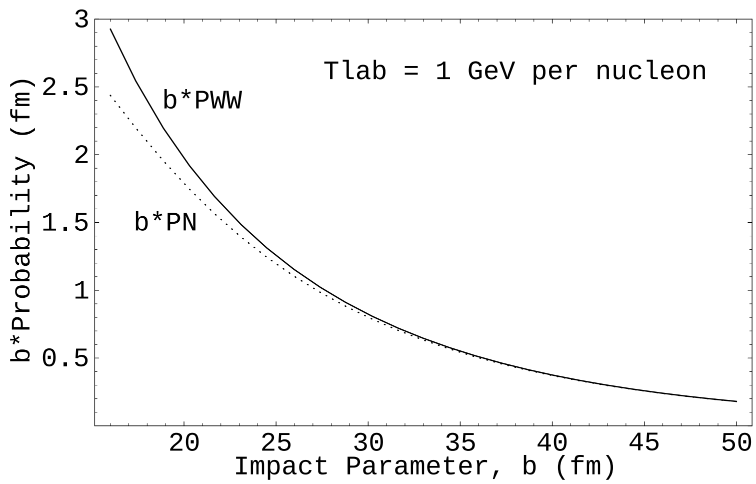

Figure 3

Same as Figure 2 except for a projectile energy of 1 GeV/nucleon.

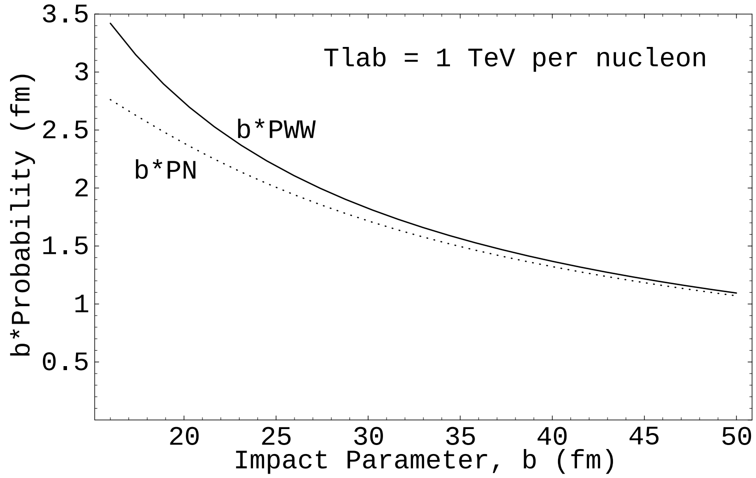

Figure 4

Same as Figure 2 except for a projectile energy of 1 TeV/nucleon.